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# The hierarchical network design problem for time-definite express common carriers

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## Abstract

Time-definite express common carriers provide time guaranteed door-to-door express service for small parcel shipments. Centers pick up and deliver parcels, while hubs consolidate partial loads. Each center is connected through a secondary route to its primary hub, while hubs are mutually connected by primary routes in a hierarchical hub-and-spoke network. The carriers may dispatch large trucks/aircraft on the primary routes but utilize smaller trucks/aircraft on the secondary routes. The time-constrained hierarchical hub-and-spoke network design problem involves determining the fleet size and schedules on the primary and secondary routes to minimize the total operating cost, while satisfying the desired level of service. We developed a route-space directed network and modeled the problem as a 0–1 binary program. An implicit enumeration method with an embedded least time path subproblem was developed. The sensitivity analysis on the service level in a partial line-haul operations network for the second largest carrier in Taiwan showed that the costs are not strictly monotonically increasing with the service levels, rather they are monotonically non-decreasing according to a step function. In addition, the determination of the sort start and pickup cutoff times has a great impact on the total cost.

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## 1. Introduction

Express common carriers (couriers) are less-than-truck load carriers, providing door-to-door delivery service for urgent parcel shipments. They provide service to, and may not discriminate against those who, called shippers, are willing to pay the published tariffs, and are thus classified as *common carriers*. Depending on the transportation modes used, they are categorized as *ground*-*exclusive* and *air-ground intermodal* carriers. The latter use the air mode for long-haul movements to cut the transit time, while the former moves shipments exclusively on the ground. As a result, the former can only serve a relatively small geographic area (Lin, 2001b). Furthermore, there are two types of time committed service. The *fixed routes and schedule* carriers publish truck (flight) schedules and always transport and deliver shipments on the next truck (flight) out. The *time-definite* carriers publish cutoff times for pickups and guarantee on time delivery.

*Hub-and-spoke networks* reduce the number of under-utilized point-to-point direct loads (Chestler, 1985). As a result, load factors are increased and total operating costs are reduced (Akyilmaz, 1994; Bryan and O'Kelly, 1999). This network configuration is widely adopted by carriers. The *pure* hub-and-spoke network, the most common, illustrated in Fig. 1, requires that all loads must either start or end at a hub sort (Bryan and O'Kelly, 1999; Eckstein and Sheffi, 1987; Leung et al., 1990; Lin, 2001a). A complete door-to-door delivery cycle in the network consists of *local service* and *line-haul operations*. Each center is the point of collection and delivery for its exclusive service area. Centers dispatch a fleet of package cars (delivery trucks) delivering shipments to consignees, and subsequently collect new shipments from the shippers. This process is local service. When new shipments arrive at the centers, the local service is completed while the line-haul operations begin.

Centers for the air-ground intermodal carriers unload shipments from package cars and reload them onto and subsequently dispatch a single tractor-trailer/package car to the primary airports. The airports act as an aggregate point of collection for pickup and delivery for their satellite centers, which are also a point of modal exchange. Centers (airports) for the ground-exclusive (air-ground intermodal) carriers run a *local* sort, where new shipments are unloaded from



Fig. 1. The delivery network in a pure hub-and-spoke network. (a) Ground-exclusive express carriers, (b) air–ground intermodal express carriers.

package cars (tractor-trailers/package cars), consolidated into a small number of full loads and reloaded onto a fleet of long-haul tractor-trailers (aircraft) for the hubs. In practice, long-haul tractor-trailers are called *feeders*. At the hub sort, the inbound shipments are unloaded from tractor-trailers (aircraft), consolidated and reloaded onto a fleet of outgoing tractor-trailers (aircraft) for the centers (airport) for local delivery (satellite centers). When unable to build full loads for the individual centers (airports), hubs make loads for other hubs for additional consolidation. On the day of delivery, centers (airports) receive their delivery volumes (aggregate volume for the satellite centers). They run a *preload* sort, at which the loads are unloaded from the feeders (aircraft), sorted and reloaded onto each package car (tractor-trailer/package car) for local delivery (each satellite center). An additional unloading, sorting and reloading onto a fleet of package cars for local delivery is necessary by each center of the air-ground intermodal carrier. This completes the line-haul operations and triggers any round of local service. Thus, hubs and centers (hubs and airports) constitute a hub-and-spoke network for ground-exclusive (air-ground intermodal) carriers. Spoke routes radiating from the hubs connect centers (airports), while interhub routes connect a pair of hubs.

Practically, there are other types of hub-and-spoke network configurations for line-haul operations. The hub-and-spoke network with stopover illustrated in Fig. 2, allows inbound-to-hub feeders (aircraft) to stop over other centers (airports) to collect additional freight along the route (Kuby and Gray, 1993). In practice, this called *tapping off*. Similarly, outbound-from-hub feeders (aircraft) may stopover at a set of centers (airports) unloading delivery freight. Stopover detours to centers (airports) along the routes may increase the transportation cost, but the reduction of some feeder routes (flights) outweighs this increase. As a result, the overall operating cost is decreased.

Those two types of hub-and-spoke network configurations place no restrictions on how many hubs, centers (airports) may feed pickups or receive delivery volume. The hierarchical hub-andspoke network configuration illustrated in Fig. 3, clusters (satellite) centers/airports around their primary hubs. Satellite centers/airports feed their pickups to and receive their delivery volumes solely from their primary hubs. Hubs consolidate the pickup (delivery) volume originating from



Fig. 2. Pure hub-and-spoke network with stopovers.



Fig. 3. Hierarchical hub-and-spoke network.

(destined to) satellite centers/airports that are destined to (originate from) other center/airport clusters.

Thus, each center/airport must connect to its primary hub through a secondary feeder/aircraft route, while each hub must connect to other hubs through a primary feeder/aircraft route. Freight from one group of origin centers/airports to a group of destination centers/airports must pass through their respective primary hubs. The freight of an OD pair, center 1 to center 6, in Fig. 3, follows route center  $1 \rightarrow \text{hub } A \rightarrow \text{hub } C \rightarrow \text{center } 6$ . The first and third segments, center  $1 \rightarrow \text{hub } A$  and hub  $C \rightarrow \text{center } 6$ , are on the secondary routes; while the second segment hub  $A \rightarrow \text{hub } C$ , is on the primary route. This network structure is common when none of the centers pick up sufficient volume to build full loads for any of the other clusters. Furthermore, while large trucks/aircraft are used on the primary routes, smaller trucks/aircraft may be used on the secondary routes. Two different sized fleets may result in a higher load factor (Kuby and Gray, 1993).

In this research, the time-constrained hierarchical hub-and-spoke network design problem (THNDP) for time-definite express carriers is studied. The goal is to simultaneously determine the fleet size and routes and schedules for both the primary and secondary feeders/aircraft so that the sum of the fixed and operating costs is minimized while meeting the desired level of service. Current et al. (1986) first introduced the hierarchical network design problem. In subsequent research, Current (1988) extended the design problem to include transshipment facilities with fixed costs at the intersections of the primary and secondary routes. However, neither research considered the time restrictions on these routes. Moreover, they required all of the hubs to be linearly connected to a single primary route. Thus, the primary route design subproblem becomes a *K*-shortest path problem, while the secondary route design subproblem becomes a minimum spanning tree problem. Lin (2001b) studied the secondary route design problem with degree and

time restrictions. In this research, we simultaneously design the primary and secondary routes that are integrally constrained by the desired level of service.

The structure of this paper is as follows. In Section 2, we propose a route-space directed network for the THNDP. In Section 3, we model the path formulation for the THNDP as an integer program. The mathematical model has an embedded least time path subproblem; therefore, we propose an implicit enumeration method for this problem. The algorithmic details are depicted in Section 4. Numerical testing on the efficiency of the algorithm with a sensitivity analysis on the service level, cutoff and hub sort start times is shown in Section 5. Our conclusions are discussed in Section 6.

# 2. The route-space directed network

The THNDP was formulated as an integer program in a *route-space directed network* (see Fig. 4). In practice, the transportation costs outweigh the handling costs in the delivery business. In addition, all freight is handled at least four times in a hierarchical network, the local sort at the pickup center and its primary hub, and the preload sort at the delivery center and its primary hub. To prevent the potential for mishandling and damage, carriers are very reluctant to plan any additional handling. Therefore, like the previous research on this subject (Barnhart and Schneur,



Fig. 4. An illustrative route-space directed network.

1996; Kuby and Gray, 1993; Lin, 2001b), in this research, the impact of fixed feeder/aircraft fleet and transportation costs on the hierarchical network design is evaluated. The node and link classifications and their associated attributes are organized in Table 1. Consider K of k center clusters, with each containing a set  $S^k$  of s centers. Moreover, there is a set H of h hubs, one in each center cluster, thus, |H| = |K|. The center local, preload and hub sorts (as described in Section 1), all have a respective node, called the *local*, *preload* and *hub sorts* in the directed network. Each node has an exogenous given sort start time and duration for consolidation. In addition, let a set  $P^k$  ( $P^H$ ) with index p, contain all *possible* secondary (primary) feeder/aircraft routes (defined in Section 4) for center cluster k (hub cluster). There are two nodes, named *arrival* and *departure*, associated with each intermediate stop on the primary and secondary routes. However, the starting and ending stops have a departure and an arrival node, respectively. Neither has an associated cost or duration.

Unloading (loading) links connect arrival (center local or hub sort) nodes to preload or hub sort (departure) nodes. The elapsed time from the arrival of feeders/aircraft to the beginning of the sort is the associated duration for the unloading links, while the sort end to the departure of the feeders/aircraft is the duration for the loading links. *Feeder/aircraft wait* links connect arrival and departure nodes to their respective stops. The associated duration is the elapsed time from the arrival to the departure of feeders/aircraft. All duration's must be long enough for unloading and loading at that stop. Otherwise, freight would only be processed in the next delivery cycle. Each feeder/aircraft wait link has an associated cost that represents a non-productive idle expenditure on the feeder driver (aircraft pilot). *Transportation* links connecting two facilities with no intermediate nodes in between have the associated transportation cost and travel time duration. The transportation cost is the sum of the proportion of driver (pilot) wages to the travel time and feeder/aircraft physical movement cost. *Freight staging* links stack the pickups or deliveries for

Туре	Function	Cost	Time duration		
Nodes					
Local	Process pickup volume	None	Consolidation		
Preload	Process delivery volume	None	Consolidation		
Hub sort	Consolidate incoming volume	None	Consolidation		
Arrival	Unload delivery	None	None		
Departure	Load pickups and transit, if any	None	None		
Links					
Unloading	Unload delivery volume (and transit if a hub sort) to preload or hub sort	None	From the arrival to the beginning of preload or hub sort		
Loading	Load pickups (and transit if a hub sort) from local or hub sort	None	From the end of local or hub sort to the departure		
Feeder/aircraft wait	Wait for unloading and loading	Crew idle time	From the arrival to the departure		
Transportation	Travel between two facilities	Crew and feeder/aircraft on road	Travel time between two facilities		
Freight staging	Stack in-hub-center pickup or delivery	None	None		

Table 1 Node and link types and attributes

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centers housed in the hub building. No costs or elapsed times are associated with these links. In summary, the route-space directed network consists of N of i nodes and A of ij links. Node duration is denoted as  $t_i$ , while  $C_{ij}$  and  $t_{ij}$  are the associated cost and duration for each link. Furthermore, there are  $|M| = \sum_k |S^k + H| * \sum_k |S^k + H|$  OD pairs with the generic element  $m \in M$ , each with an associated service commitment,  $T^m$ . Lastly, denote  $(m_0, m_d)$  the origin and destination of OD pair  $m, m \in M$ .

## 3. Mathematical model

In this research, we study the impact of fixed feeder/aircraft fleet and transportation costs on the hierarchical network design. The transportation cost is the sum of the feeder/aircraft transport cost and driver non-productive idle values. Let  $\widehat{C}(\widehat{C}^H)$  and  $C_p^k = \sum_{ij \in A} C_{ij} \delta_{ij,p}^k$  ( $C_p^H = \sum_{ij \in A} C_{ij} \delta_{ij,p}^H$ ) be the fixed and operating costs for the secondary (primary) feeder/aircraft on route p of center cluster k (hub cluster H). The path formulation of THNDP is as follows.

Objective: 
$$z = \min \sum_{p} (\widehat{C}^{H} + C_{p}^{H})h_{p}^{H} + \sum_{k} \sum_{p} (\widehat{C} + C_{p}^{k})h_{p}^{k}$$
 (1)

Subject to: 
$$t_{m_0} + \sum_i (t_{ij} + t_j) x_{ij}^m \leqslant T^m \qquad \forall m \in M;$$
 (2)

$$\sum_{i} x_{ij}^{m} - \sum_{i} x_{ji}^{m} = \begin{cases} 1; & j = m_{\rm d}; \\ -1; & j = m_{\rm o}; \\ 0; & \text{otherwise}; \end{cases} \quad \forall j \in N; \ m \in M; \tag{3}$$

$$\sum_{m} x_{ij}^{m} \leqslant B\left(\sum_{p} \delta_{ij,p}^{H} h_{p}^{H} + \sum_{k} \sum_{p} \delta_{ij,p}^{k} h_{p}^{k}\right) \quad \forall ij \in A; \ m \in M;$$

$$(4)$$

$$x_{ij}^{m}, h_{p}^{H}, h_{p}^{k} \in \{0, 1\} \qquad \forall ij \in A; \ m \in M; \ p \in P; \ k \in K$$
(5)

with decision variables:

 $x_{ij}^m = 1$ , if the freight of *m*th OD pair transverse on link *ij*;  $x_{ij}^m = 0$ , otherwise.  $h_p^k(h_p^H) = 1$ , if a feeder/aircraft is dispatched on secondary (primary) route *p* of center (hub) cluster k(H);  $h_p^k(h_p^H) = 0$ , otherwise, and parameters:

 $\delta_{ij,p}^{k}(\delta_{ij,p}^{H}) = 1$ , if link *ij* on secondary (primary) route *p* of center (hub) cluster *k*(*H*);  $\delta_{iin}^k(\delta_{iin}^H) = 0$ , otherwise.

The objective is to minimize the sum of the fixed feeder/aircraft and transportation costs for the primary and secondary feeder routes. Constraints (2) enforce the service commitments. The freight path from the local sort to the preload sort for any OD pair must meet the service commitment. Constraints (3) enforce the flow conservation. All freight must depart from the local sorts and destine to the preload sorts. Any intermediate node, feeder/aircraft arrival, departure, or hub sort, may never stage any freight. Constraints (4) are the bundling constraints, coupling freight flows and feeder/aircraft routes, which require that the freight can only be flown on the assigned primary and secondary route links. B is a big number that implies that there is insufficient demand under tight time restrictions to fill up the feeders/aircraft. All decision variables are binary as stated in constraints (5).

# 4. Solution algorithm

Without knowing the primary and secondary routes, one cannot verify whether or not the service is satisfactory. Observe that for a given  $\{h_p^{k^*}, \forall p \in P^k; h_p^{H^*}, \forall p \in P^H\}$ , the THNDP is reduced to finding a feasible solution to constraints (2)–(4) for  $x_{ij}^m$ . This involves verifying whether or not there is a time feasible path for all OD pairs. In other words, when the least time path for all OD pairs satisfies the service, a feasible solution exists. The reduced problem is a least time subproblem. For this problem we implemented the Dequeue implementation, a type of label setting algorithm (Ahuja et al., 1993).

Based on this notion, we propose an implicit enumeration algorithm for THNDP. First a search tree is constructed. This search tree consists of (|K| + 1) layers, one for each center cluster with an additional layer for the hub cluster. A travel time reduction on the primary routes will impact all of the center clusters. However, the same reduction on a cluster will only impact its' cluster. The hub cluster is therefore placed at the *bottom* of the tree to speed up the computation. Each layer consists of a set  $L^k(L^H)$  of l tree nodes, one for each *candidate feederlaircraft plan* defined as a *combination* of feasible feeder/aircraft routes for that layer (see Fig. 5).



Fig. 5. The search tree structure and branching scheme.

The rules for a candidate feeder/aircraft plan are:

- (1) Suppose there are  $P^k(P^H)$  possible feeder/aircraft routes for layer k(H), then, there are  $\sum_k \sum_{i=0}^{|P^k|-1} |P^k| C_i(|P^k|-i)! + \sum_{i=0}^{|P^H|-1} |P^H| C_i(|P^H|-i)!$  possible feeder/aircraft plans, or tree nodes.
- (2) A route is *critical* at layer k(H), whenever its exclusion will cause the service of some OD pairs to fail. Any possible feeder/aircraft plan that does not contain critical routes is excluded from the search tree.
- (3) Any possible center cluster feeder/aircraft plan that contains two routes that simultaneously serve a common center cannot be an optimal solution and is excluded from the search tree (Lin, 2001b). As an example, a plan, in Fig. 6, contains two secondary routes, Shijr → Wugu and Shijr → Wanhua → Wugu in the TPE cluster.
- (4) Similarly, any possible primary feeder/aircraft plan that contains two routes serving two hubs connected by a common transportation link is also excluded from the search tree. Substituting either one of the two routes with a less expensive route that bypasses one of the two hubs will yield a no worse feasible solution. A plan contains two primary routes in Fig. 6, Wugu → Datuen → Tainan → Nantz and Datuen → Tainan → Nantz. The solution may be improved by substituting the latter route by Datuen → Nantz.
- (5) We excluded any possible secondary feeder/aircraft plan that cannot serve all the centers in its respective cluster. We also excluded any possible primary feeder/aircraft plan that cannot service all of the OD pairs even when all of the remanding possible secondary feeder/aircraft plans are considered.
- (6) The remainder is candidate feeder/aircraft plans (tree nodes). We ordered the tree nodes for any layer using their total costs. Thus, the first tree node is the least cost plan for any layer.



Fig. 6. The optimal operating plan for the scenario 1 with ratio of 1.3.

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Initially, the existence of feasible solutions is ensured. That is, we ran the least time path code for all OD pairs with all of the primary and secondary routes contained in the candidate feeder/ aircraft plans. When there is an OD pair that fails to meet the level of service, no feasible solutions exist and the program terminates. Otherwise, the implicit enumeration algorithm begins. One tree node from each of the layers is selected to run the operations. This is called the *current combination of feederlaircraft plans*. Thus, the algorithm starts with a selection of the least cost center and hub cluster layer tree nodes. The tree node for the hub cluster is denoted as the *current cell*. This approach is a depth-first search method. Denoted  $\delta_{p,l^*}^k(\delta_{p,l^*}^H) = 1$ , if route p is in the tree node  $l^*$  of layer k(H). The total operating cost for the current combination is:

$$z = \sum_{k=1}^{|K|} \sum_{p} (\widehat{C} + C_{p}^{k}) \delta_{p,l^{*}}^{k} + \sum_{p} (\widehat{C}^{H} + C_{p}^{H}) \delta_{p,l^{*}}^{H}$$
(6)

At any current cell, there are three possible branches (see Fig. 5). (1) A time infeasible. When the current combination of feeder/aircraft plans fails to meet the time commitment for all OD pairs, we move on to the next tree node of the hub cluster. It becomes the current cell. (2) A superior feasible. We update the incumbent solution and move forward to the next tree node of the previous layer. The first feeder/aircraft plan of the hub cluster becomes the current cell. (3) An inferior feasible. Since the tree nodes are ordered by their costs in all layers, it is unnecessary to scan any higher cost tree nodes in the hub cluster. Therefore we move forward to the next tree node of the previous layer. Again, the first feeder/aircraft plan for the hub cluster becomes the current cell.

Whenever, we encounter the *edge*, we move one layer backward. If necessary, the process continues, until the next tree node in a layer has not yet been scanned. When determined, the first tree node of all of the layers downward with the first feeder/aircraft plan for the hub cluster as the current cell are in the current combination for evaluation. If no node exists, the incumbent is the optimal solution and the program terminates.

## 5. Computational results and sensitivity analysis

The second largest ground-exclusive same-day express common carrier in Taiwan provided data for numerical testing. This carrier provides repetitive hourly services. A high service and operation frequency increases customer satisfaction and reduces facility and feeders per trip operating costs. This carrier has divided its service territory, Taiwan island, into 13 express districts (center clusters). Each district designates a center as the hub to serve the other centers in the district. Hourly, satellite centers forward their pickups using a fleet of 3.5-ton feeders to their primary hub for consolidation. These are *secondary* feeder routes. A fleet of 10.5-ton feeders connects all of the hubs. These are *primary* feeder routes. In this research, the top four express districts, Taipei (TPE), Taichung (TCH), Tainan (TNN), and Kaohsiung (KSG), and some of their major satellite centers were chosen to form our test network (see Fig. 6). The default service level for centers in the same district, and between TPE-TCH, TCH-TNN/KSG, and TPE-TNN/KSG were set at 2, 5, 5 and 7 h. The unit transportation cost for secondary and primary feeder routes was NTD\$2.51 and 3.19 per km, while the driver per minute wage cost was NTD\$3 (monthly salary of \$31,680). The feeder fixed cost was NTD\$182.8 for both fleets, including

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1.771

depreciation, licensing taxes and salaries. At each stop, 5 min was allowed for either unloading or loading.

This program was coded in C and run under a Linux O/S on a PC equipped with a Pentium III-500 Mhz chip. The minimum service time required for centers between TPE-TCH, TCH-TNN/ KSG, and TPE-TNN/KSG, an output from the initial least time program, were 5.2, 5.2 and 7.2 h, respectively. In addition, a sensitivity analysis was performed on the service level, the cutoff and hub sort start times. The service level range varied from 1.1 to 1.6 times the base service commitment. Moreover, the cutoff and hub sort start times were varied and created a total of three operational scenarios. In scenario 1, all of the times were set at the hour on the hour. Scenario 2 was the same scenario 1, except that the hub sort start time at the TPE cluster was at the halfhour. In scenario 3, all of the times, cutoff and hub sort start, in the TPE cluster were set to the half-hour while maintaining all others the same. The computational results are organized in Table 2.

Overall, the results are encouraging. Except for a few cases (16.7%), the computing time for all the scenarios in this network was no more than 20 s. The critical routes play a crucial role in the overall computational efficiency. For two identical optimal operational plans, 1.2 and 1.3 times of the base service level in scenario 1, the time was reduced from 8891.15 to 4.42 s.

We made the following observations about the empirical analysis. When the service level is low (long), only one route serves all of the centers for each cluster and the hub cluster. The computational time was quite fast, since they are the first tree node in each layer. When the service level

Sort time	Ratio to base service	Feeder	Feeder routes					Total	Iterations		CPU (s)
		Second	Secondary				Primary Critical	cost	Opt.	Total	
		TPE	TCN	TNN	KSG			(NTD\$)			
1	1.6	1	1	1	1	1		4143	1	2	4.31
	1.5	1	1	1	1	1		4143	1	2	4.30
	1.4	2	1	1	1	1		4365	12,157	12,158	1088.65
	1.3	2	1	1	1	2		5616	99,320	99,323	8891.15
	1.2	2	1	1	1	2	1	5616	1	4	4.42
	1.1	2	1	1	1	3	3	8693	3	21	17.74
2	1.6	1	1	1	1	1		4143	1	2	4.89
	1.5	1	1	1	1	1		4143	1	2	5.00
	1.4	1	1	1	1	1		4143	1	2	4.83
	1.3	1	1	1	1	2		5394	1	4	4.98
	1.2	1	1	1	1	2		5394	1	4	5.09
	1.1					Infea	sible				0.01
3	1.6	1	1	1	1	1		4143	1	2	4.71
	1.5	1	1	1	1	1		4143	1	2	4.71
	1.4	1	1	1	1	1		4143	1	2	4.75
	1.3	2	1	1	1	1		4365	12,157	12,158	1089.17
	1.2	2	1	1	1	2	1	5616	1	4	4.83
	1.1	2	1	1	1	2	1	5616	1	4	4.83

### Table 2 Computational results



Fig. 7. The sensitivity analysis on service level.

gradually improved, more routes were required to serve the tighter service commitment. Adding secondary routes is cheaper than adding primary routes. Thus, the optimal operating plans will gradually add secondary routes until additional primary routes must be included to meet the desired level of service.

The service level impact on the total operating cost does not have a *strictly* monotonically increasing relationship. It is a monotonically non-decreasing function with several plateaus, as shown in Fig. 7. The most obvious plateau is in the range of over 1.4 for all of the three scenarios. This means that the carrier may improve its' service level without incurring any additional costs. The longer the distance between clusters the higher the impact on the overall operating cost. A stiff increase from the ratio 1.4 to 1.3 in scenarios 1 and 2, and also 1.3 to 1.2 in scenario 3 represents that an additional primary route must be introduced to meet the service between the farthest northern and southern districts.

The sensitivity analysis on the cutoff and hub sort start times, also shown in Fig. 7, also demonstrated a great impact on the overall cost. The average operating cost for scenario 1 was 14.5% higher than that for scenario 2. Even though they are not as great as the hub sort start times, the cutoff times do show some slight impact on the overall cost. The average operating cost for scenario 2 is 3.5% higher than scenario 3, excluding one case of service infeasibility.

## 6. Conclusions

Time-definite express common carriers design a time constrained hierarchical hub-and-spoke network to provide time-definite door-to-door service for urgent shipments for shippers. The problem is to determine the fleet size for both the primary and secondary feeder routes and their schedules with minimal cost while meeting the desired service level. We proposed an implicitly enumeration algorithm with an embedded least time subproblem to manage the special structure of this 0–1 binary problem. The numerical tests showed some encouraging results that demonstrated that this approach is a suitable planning method for the design of a carrier operations network. The sensitivity analysis on the service level showed a quite interesting result. The operating cost with respect to the service level does not possess a strictly monotonically increasing

function, but rather a monotonically non-decreasing function with several plateaus. This means that the carrier can improve its service levels without incurring additional operating cost. Moreover, the planning for cutoff and hub sort start times also have a profound impact on the operating cost.

In this research, we assumed that feeder/aircraft might accommodate the current light pickup volume. However, when the volume grows continuously, this may become a critical restriction. The THNDP then becomes a time constrained HNDP in a capacitated network. The current approach can be modified accordingly. That is, the subproblem becomes a least time problem in a capacitated network. One may implement a *K*-shortest path code for this subproblem.

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